Multiresolution Elastic Matching

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Abstract

This summary provides a general overview of the work Multiresolution Elastic Matching \[1\] written by Ruzena Bajcsy and Stanislav Kovačič.

1. Introduction

This section provides a general introduction of the topics presented in \[1\] and provides some additional background which is important in order to understand the paper. In section 2 the general matching process is introduced and discussed. Section 3 provides more information about the implementation and its details. In section 4 several experiments are presented to illustrate some results of the presented paper. This summary ends with a conclusion presented in section 5.

1.1. Goals

The general idea behind the work presented in \[1\] is to develop a mechanism to match "an explicit 3-dimensional pictorial model to locally variant data". In other words, two existing representations (one model representation and one input representation) are put into correspondence. To achieve this goal, rigid transforms are used to account for global misalignments, while elastic deformations are applied for local shape differences. A coarse-to-fine strategy (called multiresolution elastic matching) is the heart of the presented paper and leads to more efficient computations and improved convergence of the matching process in general.

2. The Matching Process

In the presented paper two virtual objects are given: One reference object and one made out of an elastic material. To mimic manual registration, larger disparities are corrected in a first step. The second step consists of making improvements concerning smaller disparities. This second step is then repeated until a "satisfactory" match is achieved. The input is given by brain scans gained from computed tomography (CT). The virtual model is a voxel representation of an anatomical human brain atlas.

2.1. What needs to be defined?

In general, the presented algorithm aims to find a mapping \( T \) from a given model \( M \) (i.e. the brain atlas) to a pattern \( P \) (i.e. the input CT brain scans) using a similarity measure \( S \). The similarity measure (or similarity function) will be discussed in more detail in 2.3.3. The following questions have to be answered to help the algorithm produce satisfactory results:

- What should be matched? (i.e. which features should be used in matching?)
- Which constraints should be considered?
- How should be matched? (i.e. which matching process for achieving a consistent match should be applied?)
- How should the match be evaluated? (i.e. how should the similarity measure be defined?)

2.2. Global Matching

As discussed in 2.3, the elastic matching algorithm is a good choice for smaller changes but it fails if global misalignment is too large. To account for larger global misalignments global matching is applied first. This algorithm tries to eliminate the effects of the following three mayor types of misalignment:

- Translational misalignment
- Scaling misalignment
- Rotational misalignment

Fig.1: Translational misalignment

Source: en.wikipedia.org
As seen in Fig.1, translational misalignment depicts the case in which the objects are (far) away from each other. In Fig.2 it can be seen that scaling misalignment means that one object is bigger than the other, while Fig.3 shows that rotational misalignment describes the case in which one object is rotated in a different direction than the other. In order to eliminate these misalignments the global matching algorithm proceeds in the following way: Firstly, each object is approximated by "an ellipsoid-like scatter of particles uniformly distributed in space". Translational misalignment is then eliminated by aligning the centers of masses of both objects. In order to eliminate rotational and scaling misalignments, the method of principal axes is used. The covariance matrix is computed for each object and an attempt is made to equalize the matrices through rotation and scaling.

Fig.2 gives an intuition how the corresponding eigenvectors and eigenvalues of two different covariance matrices behave. The left image shows the orientation of the point cloud for matrix 1, while the right image depicts the orientation for matrix 2.

\[
\begin{pmatrix}
5 & 0 \\
0 & 1 
\end{pmatrix} \quad (1)
\]

\[
\begin{pmatrix}
1 & 0 \\
0 & 5 
\end{pmatrix} \quad (2)
\]

2.3. Elastic Matching

The basic approach for eliminating small local disparities by using elastic deformations goes back to the Doctoral dissertation of Broit. In it an algorithm is described, which deforms an object until an equilibrium between external forces and resisting internal forces is achieved. This algorithm is based on a partial differential equation, which has been developed by the French scientist Navier in 1822. This partial differential equations reads as follows:

\[
\mu \Delta u_i + (\lambda + \mu) \frac{\partial \theta}{\partial x_i} + F_i = 0 \quad (i = 1, 2, 3) \quad (3)
\]

\[
u = (u_1, u_2, u_3)^T \quad \text{stands for the sought displacements and} \quad \theta = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} \quad \text{is the cubical dilation.} \]

The cubical dilation is defined as the quotient of change in volume over the size of the original volume and thus hints at the change in volume due to deformation forces. The parameter \( x = (x_1, x_2, x_3)^T \) describes the coordinate system before any deformation has taken place, while \( F = (F_1, F_2, F_3)^T \) stands for the external forces distributed in the body. The constants \( \mu \) and \( \lambda \) define elastic properties of the body. These elastic properties are an integral part of the description of the corresponding material of each object and are described in further detail below.

2.3.1 Elastic Properties

As described above, the choice of the elastic properties constants is crucial for a proper modelling of each objects material. The authors of this paper decide to set the parameter \( \lambda \) to zero in order to be left with only one elastic constant. This single constant \( \mu \) is then responsible for modelling the material. Experiments indicate that a large value of \( \mu \) leads to more rigid materials (which might, for example, resemble rubber), while a smaller value of \( \mu \) leads to a solution, which is mostly controlled by external forces. One disadvantage of choosing a very small value for \( \mu \) is the fact that in this case noise becomes more visible and false matches occur more frequently. Thus, the authors hint at deforming the object step by step as this "should be safer".
2.3.2 External forces

An important part of the elastic matching model is where and how the external forces are applied to obtain a consistent deformation as they are used to guide the solution locally and globally. The external forces can be obtained by:

- information from input data
- information from interactive input
- information from a knowledge base
- other processes...

The major task of the external forces is to bring similar regions in both objects into correspondence. To achieve this, a similarity measure (as described in 2.3.3) between a certain position \( x = (x_1, x_2, x_3)^T \) in the first object and another position \( x + u = x + (u_1, u_2, u_3)^T \) in the second object is defined. Note that the second position already includes a displacement \( u \).

2.3.3 Similarity measure

The similarity measure or similarity function at position \( x \) is denoted by \( S(u) \). The best local match is expected for a displacement vector \( u \) which maximizes \( S(u) \). If a maximum of \( S(u) \) exists, it is also possible to increase the similarity by applying a force proportional to the gradient vector of \( S(u) \).

If it is assumed that \( S(u) \) has continuous second derivatives, it can be approximated by the following Taylor series:

\[
S(u + \delta u) = S(u) + \delta u^T g(u) + \frac{1}{2} \delta u^T H(u) \delta u \tag{4}
\]

As long as only small deformations are taken into account, a quadratic approximation of \( S(u) \) can be used:

\[
S(u) = \frac{1}{2} u^T Au + b^T u + c \tag{5}
\]

If this is substituted into the partial differential equation 3 mentioned above, one can get a description for the applied external forces:

\[
F(u) = g(u) = Au + b \tag{6}
\]

The full notation of this equation reads as follows:

\[
F_i = 2a_{i1}u_1 + 2a_{i2}u_2 + 2a_{i3}u_3 + b_i \quad (i = 1, 2, 3) \tag{7}
\]

Those forces are applied in regions in which \( S(u) \) has a maximum. Other regions are not used for matching as they lack reliable matching information.

2.3.4 Multiresolution Elastic Matching

As already mentioned above, a multi-resolution strategy is applied for elastic matching. This strategy has been inspired by multigrid techniques used in numerical mathematics [6], computational vision [7] [8] and multiresolution picture processing [9]. The idea is similar to image pyramids for which Fig.5 shows an example.

Fig.5: Sample image pyramid

Source: [http://vis.berkeley.edu](http://vis.berkeley.edu)

The basic idea behind such an image pyramid (according to [10]) is to create a series of images which are weighted and scaled down. When this technique is used multiple times, it creates a stack of successively smaller images, with each pixel containing a local average that corresponds to a pixel neighbourhood on a lower level of the pyramid. Fig.6 shows the scheme which has been deployed for the algorithm mentioned in [1].
The image caption of Fig.6 in the original paper reads as follows: "A multiresolution deformation scheme. Features obtained from the resolution pyramid for the model M and the data D are entered into [the] elastic matcher. Matching starts on a coarse resolution level. The interpolated solution from coarser level is entered as the first approximation to the next finer level. The finest level solution is used to incrementally deform the model until a satisfactory match is achieved."

3. IMPLEMENTATION

After discussing the general ideas of the presented paper, this section gives some more details about the implementation. The general steps of the matching algorithm are:

1. Preprocessing of the input CT brain scans
2. Preparing the brain atlas for matching
3. Matching the brain atlas with given CT brain scans
4. Overlaying the brain atlas with CT brain scans after matching

In the following the results of these steps are to be discussed.

3.1. Preprocessing

Real CT scans of normal patients were used for the following experiments. It was necessary to process these scans first to obtain usable data.

Preprocessing of the input CT brain scans includes optional low-pass filtering, resampling of the slices, interpolating by linear interpolation to get cubically shaped voxels, selecting an 8-bit grey value window in the original 12-bit range, removing the skull to isolate the brain in CT scans and also the optional removal of large calcifications (white spots) if present. Fig.7 shows the result of removing the skull in order to isolate the brain.

3.2. Preparing the brain atlas

The purpose of the brain atlas is to provide a 3-D model of a healthy (human) brain. Fig.8 shows a brain atlas from 1976, which has been created by Robert Livingston at the University of California, San Diego.

3.3. Matching and overlaying

After preprocessing and preparing the brain atlas, global matching is performed as described in 2.2. After this procedure an overlap of roughly 80% between the brain atlas and the CT brain data is achieved. If this overlap is visualized by contour lines (representing the brain atlas) superimposing the input data, a result as seen in Fig.9 (left) can be expected.

After global matching, elastic matching as described in 2.3 is performed. The result of the latter procedure is depicted in Fig.9 (right). Two features are being used for matching: The outer edge of the brain and the brain ventricles. Other anatomical structures are deformed as a side-effect of ventricle and outer edge matching because deformations propagate through the elastic matching process. In the left column of the image below the average grey value can be seen while the other three columns show the corresponding changes of the features in $x$-, $y$- and $z$- direction.
4. Experiments

The following experiments are meant to give the reader some more intuition about the discussed ideas and algorithms.

4.1. Elastic constants

As already seen in 2.3.1, elastic constants play a big role in determining which properties a modelled material has. Hence, they are also very important for the elastic matching process in general as Fig. 10 (left) shows vividly. The left column depicts the matching state after performing global alignment. The three columns to the right show matching results for an elastic constant value of 8, 4 and 1 respectively.

Fig. 10: Matching between two brain atlases using different elastic constants (left) and deformation at successive stages (right)

To show that not only the elastic constants but also the number of iterations of the elastic matching process influences the final result, Fig. 10 (right) illustrates the result of the matching process with a fixed elastic constant of value 2. The first column shows the result after global alignment while the next three columns to the right depict the result after successive deformations. It can be easily seen that better results are achieved with more iterations.

Fig. 11: Two brain atlases superimposed after global alignment (left) and elastic matching (right)

In Fig. 11 it can be seen that it is also possible to match two brain atlases with each other. The left picture illustrates the matching result after global alignment and the right picture the result after elastic matching. Fig. 12 (left) hints at the possibility of matching the brain atlas to different brains. In this example each row shows a different CT brain. Lastly, Fig. 12 (right) presents all matching ideas of this paper by showing one brain atlas slice after global matching, elastic matching and scaling it back to the original CT pixel size.

Fig. 12: (Elastic) matching the brain atlas to three different brains (left) and example including all presented matching steps (right)

5. Conclusion

The main contribution of the presented paper is the development of a general method for matching 3-D patterns with a 3-D model. To achieve this goal, rigid transforms are used for global misalignment and elastic deformations are used for local shape differences. A coarse-to-fine strategy approach is applied which leads to more efficient computations and improved convergence of the matching process in general. The matching algorithm is performed in 3-D.

References